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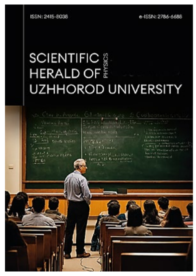

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Interdisciplinary application of probability theory and mathematical statistics in professional orientation for physicists and mathematicians

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Abstract

Relevance. Mathematical statistics methods can play a valuable role in educating students by assessing their mathematical abilities, guiding professional orientation choices, and evaluating potential solutions.

Purpose. The purpose of the study is to explore the possibility of using ubiquitous distance education for collaborative group work with mathematical support. The proposed approach involves the use of cluster and network technologies, where each participant acts as a host for the curriculum.

Methodology. The methodology involves determining the effectiveness of the applied methods by evaluating the potential for group application of the methodological apparatus and identifying areas of assistance for students in choosing a professional orientation.

Results. The results demonstrate that in higher education, the use of mathematical tools enables students to develop decision-making abilities, assess choice conditions, select courses, and identify knowledge domains where they can thrive. For mathematics students, this approach enhances their ability to apply mathematical concepts to practical problems through collaborative group work.

Conclusions. The practical significance of the research is determined by the potential of the developed program to shape professionally trained graduates.

Keywords: university; probability theory; statistics; education; profession.

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Introduction

An important issue of modern pedagogy of higher education is the theoretical and methodological support for the development of such a form of education as distance learning, which is determined by the spread of information and communication technologies and the intensification of their use in educational practice, additional opportunities for individualising instruction, for implementing the principles of accessibility, visibility, and "life-long education" [1]. The solution to this problem can be associated with a number of research areas, the most important of which, in our opinion, are: the willingness of teachers to introduce distance learning technologies, which requires them to have a high level of information and pedagogical competence; willingness of students and adults to use this form of knowledge and skills [2]. The latter requires self-organisation, high motivation, and is closely related to perception and attitude to distance education [3]. Therefore, the study of the value judgments of potential consumers of educational services is useful for the pedagogical community to identify ways to improve and disseminate distance education [4]. In a crisis state of the economy, this is determined by the fact that distance learning, in fact, forms a holistic perception among students [5].

Considering the place and role of distance education in ensuring the development of human capital, and, accordingly, the socio-economic and socio-political subsystems of society, the corresponding phenomenon is increasingly positioned within the framework of modern scientific discourse, including beyond the scope of pedagogical scientific thought [6]. Furthermore, the problems of distance education are mainstreamed in the context of the content of state programmes and measures for the development of the educational sector. Thus, education development strategies among the main directions of its implementation necessitate the following [7]:

– settlement of the organisation of education in distance learning;

– creation of regional centres and distance learning systems.

Given the importance of each of these areas for the further development of both the content of distance learning in general, and for improving the practice of its direct use in the professional activities of educational institutions, "distance learning in the training of future specialists (by industry)" and "distance and dual higher education" are determined at the level of urgent problems within the framework of priority areas of scientific research [8]. In these areas, the issue of assessing the strength of manifestation of the individual determinants of distance education is of importance both for improving the national policy for the development of the appropriate form of higher education (state level), and for increasing the efficiency of the use by higher education institutions of the potentials functioning based on modern psychological, pedagogical, and information and communication technologies of a specialised environment (university level) [9]. The presence of knowledge in the developmental subjects of the remote form of higher education on the potential impacts of various factors on the effectiveness of its use in the educational process will not only specify the objects of managerial impact, but also take measures to adjust the volume of use of organisational resources [10]. In other words, the definition (clarification) of the manifestation of the determinants of the development of distance education is not only relevant, but also practically meaningful considering the requests from the subjects of educational management [11].

Literature review

The national policy of most countries of the world that have a stable level of development of socio-political and market institutions to ensure the quality of education and the development of its content has long been focused on the formation and use of the potentials of distance learning [12]. The distance in training not only allows to expand the scope of users and applicants for educational and information resources, but also becomes the reason for the mainstreaming of certain scientific and practical tasks, for example, those tasks, the content of which is associated with the assessment of motivation and readiness of distance education subjects for the active use of tools of information and communication technologies to improve the achievement of educational content goals [13]. The issues of the use of information and communication technologies in the system of organising the educational process as part of the distance form of higher education constitute the subject of scientific attention of Kazakh and foreign scientists [14]. Over the past five years, "distance learning progress" has covered all continents, even South Asia and Africa have been viewed in the context of equal access to higher education by 2030 [15]. The vast majority of scientific research in theory and practice of the use of distance learning was focused on the investigation of the issues of the effectiveness of distance education and determining the significance of some of its determinants within the framework of the final quality of the result of educational and scientific activities [16]. The vast majority of studies carried out by foreign scientists focused on determination of the criteria for the quality assessment of distance education, as well as on identification of factors influencing the level of interest of potential consumers of educational services for the use of various distance learning tools [17].

Despite the sufficient attention of scientists to the problems of the use of distance learning in higher education, some of its questions are still open for further scientific research [18]. Among these issues, of extraordinary importance are those which focus on the elucidation and analysis of the opinions of participants in educational and scientific communication regarding the determinants of the development of distance education [19]. Determination of the strength of manifestation of the relevant determinants will allow the subjects of performance and development management of the higher education system not only to prioritise the use of various public administration mechanisms for higher education (state level), but also to increase the efficiency of interaction between subjects and objects of educational communication (university level) [20].

Materials and Methods

Modern distance education is rapidly spreading throughout the world. Its development tendencies are associated with commercialisation, an increase in the possibilities of obtaining both educational online levels and other types of online education, the search for means of improving the quality of distance learning and the reliability of monitoring its results. The distance learning system in the countries of the former USSR is still only in its infancy, however, subject to the use of world experience, the combination of advanced distance education technologies with the best technologies and methods of classical forms of education, it can have many prospects.

The authors investigated the historical background of the affirmation of distance learning at the international and state levels, showed the legitimacy of its use to obtain degrees and competencies, improving the level of training of specialists, including through the formation of an environment in which all students interact with each other. Moreover, their understanding of the subject is determined by the fact that they work in a group.

The methods applied in the paper are mainly mathematical modelling and analogy. The conditions that form the student's access to the educational environment shown the main value. We believe that such access can be based on the understanding that the teacher provides not only the material and, accordingly, explanations to it, but above all the structure that allows to ensure the level of mastering on the part of the student. This is similar to the formation of a network structure, which fully affects the aspect of the passage of knowledge or the amount of information through the student network [21].

The study classifies and lists the advantages and disadvantages of distance education, which include organisational and pedagogical, information technology, and psychological. The authors identified the problems of introducing distance education at the university and, accordingly, the strategic and tactical prospects for its development. According to the results of the analysis of respondents' opinions regarding the assessment of the power of manifestation of certain determinants (factors) of the development of distance education, the potentials of their influence on the level of popularisation of the corresponding form of higher education among subjects and objects of educational programs are determined. Depending on the level of significance of the influence of a particular factor on the dynamics of the development of distance education, their hierarchy is compiled (factors are arranged in order of reduction of their influence potential):

– individual features of the applicant for higher education;

– the unwillingness of stakeholders to equate the quality of higher education received as part of the full-time (daytime, evening) form of education with the quality of higher education received within the framework of distance learning;

– imperfection of the institutional environment for the implementation of universities and distance learning;

– imperfection of the content, forms and technologies of the implementation of the distance form of education;

– the absence of those educational programmes that are the most demanded by the labour market among those offered as part of the distance learning course.

Despite the hierarchy of determinants proposed above for influencing the development of distance education, we cannot talk about the secondary nature of those that are at the end of the list. Each of these determinants has its own potential for influencing the development of the distance form of higher education, therefore, it must be identified at the level of the object of direct attention from the subjects of educational management. The decision on the development of distance learning in higher education, as well as its popularisation among potential applicants, falls within the competence of universities and the state, and the influence of the latter on the effectiveness of solving relevant issues is decisive. The promising areas of work of specialised government bodies, as well as university administrations, in the development of distance learning include:

– creation of favourable conditions for the development of a state of readiness of a person to use the information and communication technologies of distance learning to acquire knowledge in the higher education system (for example, through the use of distance learning technologies in the secondary education system);

– improvement of the content of institutional norms to ensure the functioning of distance learning (for example, through the formation and implementation of a public procurement for the training of specialists with higher education in the distance learning system; the establishment of mandatory volumes, at least for those universities that have research status, in training specialists with the use of distance learning as a separate form of training, etc.);

– popularisation of the level of professionalism of those individuals who have received higher education with the use of information and communication technologies of distance learning (for example, the state may introduce temporary benefits for those employers who will offer jobs for specialists whose professional knowledge and skills were formed with the use of distance learning, etc.).

The dynamics of the development of distance learning depends, on the one hand, on the ability of its higher education system to offer competitive professional knowledge and an effective mechanism for relaying them with the use of the information and communication technologies of distance learning, and on the other hand, on the individual willingness of potential higher education applicants to use the potentials of distance learning for gaining professional knowledge, as well as from the willingness of employers directly and society at large to recognise the equivalence of knowledge and skills acquired with its help to those competences that can be formed within the framework of conventional forms of training.

Results and Discussion

To solve the problems of clustering a distributed network of students, the authors propose application of the well-known clustering algorithm of k -means with certain improvements.

The $k-means$ algorithm is one of the algorithms that solves clustering problems. This algorithm is a non-hierarchical iterative clustering method, which has gained great popularity due to its simplicity, visualisation of implementation, and fairly high quality of work. The main idea of the k-means algorithm is that the data is randomly divided into clusters, after which the center of mass for each cluster obtained in the previous step is recalculated by interactivity, then the vectors are again divided into clusters according to proximity of the new centers to the selected metric.

The purpose of the algorithm is to divide n observations into k clusters so that each observation belongs to only one cluster located at the smallest distance from the observation. Euclidean distance is used as a measure of proximity (1):

$$p(x,y) = x - y = \sqrt{\sum_{p=1}^{n}\left(x_p - y_p\right)^2}, \qquad (1)$$

where $x, y \in R^n$.

The authors consider the series $X = \{x_1, x_2, \ldots, x_n\}, x_i \in R^d, i = 1, \ldots, n$ with n observations. The k-means algorithm splits X into k sets $S_1, S_2, \ldots, S_k$, so as to minimise the sum of the squares of the distances from each point of the cluster to its center (the centre of cluster masses). The authors introduce the notation $S = \{S_1, S_2, \ldots, S_k\}$. Then the action of the k-means algorithm is equivalent to that of search (2):

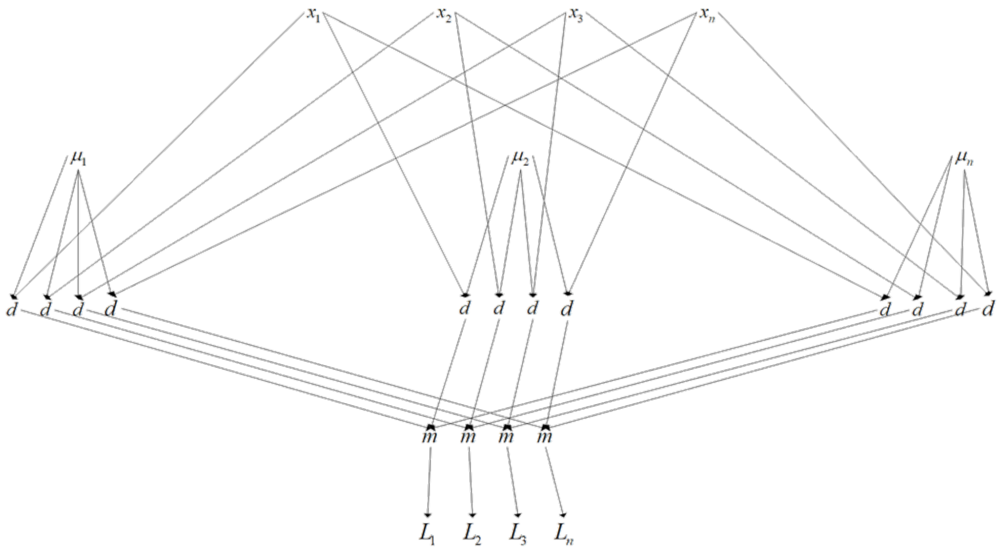
$$\underset{S}{argmin} \sum_{i=1}^{k} \sum_{x \in S_i} p(x, \mu_i)^2, \qquad (2)$$

where μ_i – cluster centres, $i = 1, \ldots, k, p(x, \mu_i)$ – distance function between x and μ_i.

Algorithm steps:

1. Cluster initialisation. An arbitrary set of points $\mu_i, i = 1, \ldots, k$ is chosen, which are considered as the initial cluster centres $\mu_i^{(0)} = \mu_i, i = 1, \ldots, k$.

2. Cluster distribution of vectors (Fig. 1). For each t iteration $\forall x_i \in X, i = 1, \ldots, n: x_i \in S_j \Leftrightarrow j = \underset{k}{argmin} \rho\left(x_i, \mu_k^{(t-1)}\right)^2$ is selected. The distribution of vectors among clusters involves the calculation of the distances between each vector $x_i \in X, i = 1, \ldots n$ and the cluster centres $\mu_j, j = 1, \ldots, k$ (Fig. 2). Thus, this step provides kn calculations of the distances between d-dimensional vectors.



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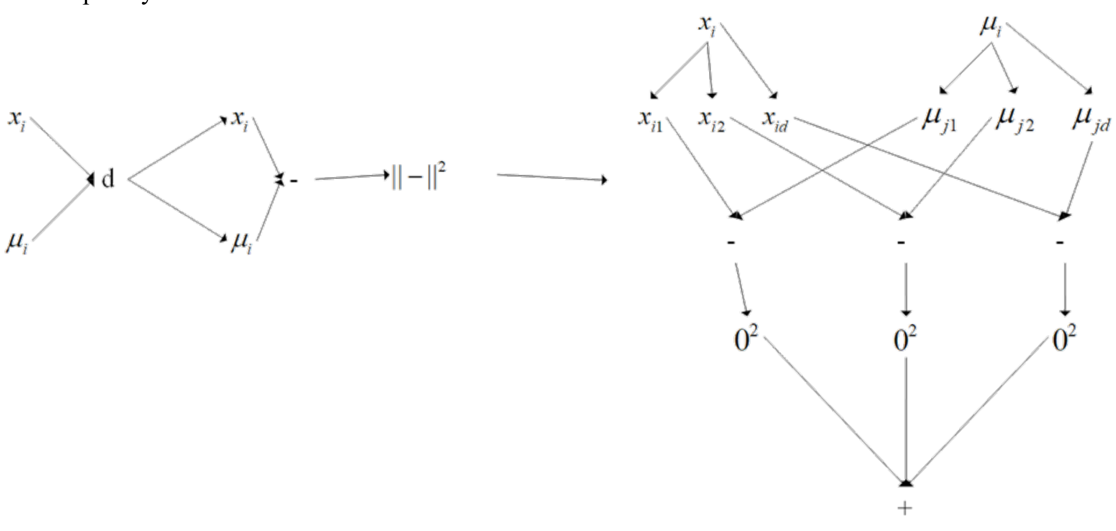

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3. Recalculation of cluster centers. For each t iteration $\forall i = 1, \dots k: \mu_i^{(t)} = \frac{1}{|S_i|}\sum_{x \in S_i} x$. If $\exists i \in \overline{1,k}: \mu_i^{(t)} \neq \mu_i^{(t-1)}$, then authors point to proceed to re-checking the cluster centre, otherwise the algorithm stops its work. Thus, the cluster centre is found.

Recalculation of cluster centres involves k calculations of centres of masses μ_i of the sets $S_i, = 1, \dots, k$ (Fig. 3).

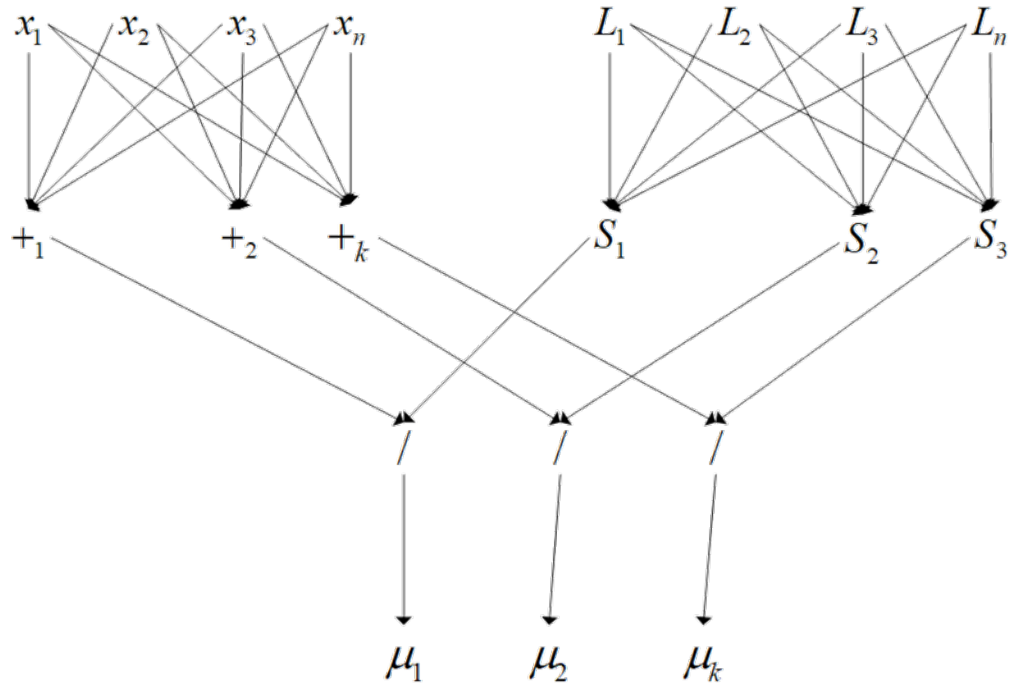

Figure 3. Cluster centre recalculation scheme

To initialise the centres of mass $\mu_1, \dots, \mu_k$ the authors use the method of random partitioning. For each vector $x_i \in X, i = 1, \dots, n$, a cluster $S_1, S_2, \dots, S_k$ is randomly selected and the value $\mu_1, \dots, \mu_k$ (Euclidean distance) is calculated.

The authors calculate $\Theta_{centroid}^{d,m}$ – the time complexity of calculating the centroid of a cluster, the number of elements of which is m in d-dimensional space, and $\Theta_{dis\,tan\,ce}^{d}$ – the time complexity of calculating the distance between two d-dimensional vectors.

To initialise the centres of k clusters with cardinality m in d-dimensional space, the following is identified (3):

$$\Theta_{init}^{k,d,m} = k^{*}\Theta_{centroid}^{d,m} \quad (3)$$

Fig. 4 shows a block diagram of the k-means algorithm. Due to the constant increase in the volume of information flows and the number of virtual nodes in the cloud, it is difficult to establish the most optimal route between the source and destination nodes. As an indicator for determining such a route, there may be a load, data transmission time, bandwidth, etc. In most cases, routing is considered in the Euclidean space, for example, the mathematical apparatus of this space is described in the structure of the distance learning form. Cases of communication channel overload, erroneous settings of network elements, etc. form structural heterogeneities, as a result of which the necessary route is not established, therefore, the data transfer process is complicated. To move from Euclidean space to hyperbolic, and vice versa, Ricci flows are used that deform the Riemann metric while preserving the properties of the network. To implement this, a metric tensor is needed, with the help of which it is possible to determine such important parameters as the Christoffel symbol, the Riemann and Ricci tensors, and the space curvature scalar. The authors suggest to consider an n-dimensional coordinate system. Let the various network parameters, for example, the load between nodes, act as components of such a system. The number of two-way connections between nodes at the virtual network level is defined as $n_{max} = m(m-1)$, where m is the number of nodes. If there is no load on some connections between nodes, the authors denote them by w, the system will comprise $n = n_{max} - w$ coordinates. The centre of the coordinate system O corresponds to the state of the network when $w = n_{max}$.

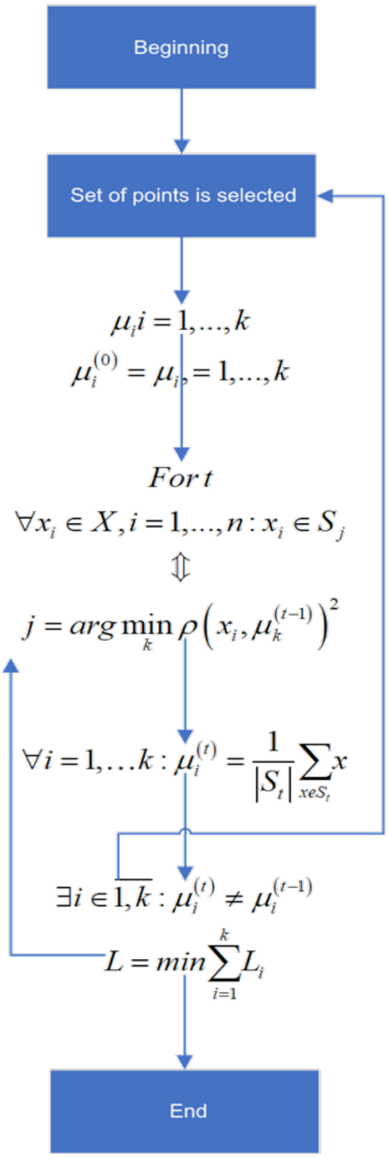

Figure 4. Block diagram of an advanced algorithm for clustering nodes of a self-organised network
Source: developed by the authors

The authors introduce the concept of radius vector $\vec{r}$, which connects the origin of the coordinate system O with a point corresponding to the state of the network (4):

$$\vec{r} = x^i \vec{e}_i = x_i \vec{e}^i, \quad (4)$$

where x_i and x^i – covariant and contravariant components, respectively; $\vec{e}_i$ и $\vec{e}^i$ – covariant and contravariant unit vectors interconnected by the correlation (5):

$$\vec{e}^k \vec{e}_m = \delta^k_m, \quad (5)$$

where δ – Kronecker symbol (tensor).

The correlation between covariant and contravariant components is carried out with the use of the following formulas (6-8):

$$x^i = g^{ij} x_j; x_i = g_{ij} x^j, \quad (6)$$

where g^{ij} – metric contravariant tensor; g_{ij} – metric covariant tensor.

$$g^{ij} = \vec{e}^i \vec{e}^j, \quad (7)$$

$$g_{ij} = \vec{e}_i \vec{e}_j, \quad (8)$$

The network state can be represented simultaneously in a covariant and contravariant coordinate system in a quadratic form, factoring in (4) and (5) in the form (9):

$$\vec{e}^2 = x^i x_i = g_{ij} x^i x^j = g^{ij} x_i x_j. \quad (9)$$

The change in load between nodes occurs systematically, therefore, a change in the state of the network is observed, which, factoring in (9), can be represented as (10):

$$(d\vec{r})^2 = dx^i dx_j = g_{ij} dx^i dx^j = g^{ij} dx_i dx_j. \quad (10)$$

It should be noted that $\vec{e}_i$ and $\vec{e}^i$ are also constantly changing, and this can be represented as (11):

$$\vec{e}_i = \frac{\partial \vec{r}}{\partial x^i} = \partial_i \vec{r}; \vec{e}^i = \frac{\partial \vec{r}}{\partial x_i} = \partial^i \vec{r}. \quad (11)$$

When the load changes, it is redistributed in the channels, which is described by the connectivity coefficient (Christoffel symbol) of the first Γ_{ij}^{k} of the second $\Gamma_{k,ij}$ kind, which in the general case is written as follows (12):

$$\Gamma_{ij}^{k} = \vec{e}^{k}\partial_i\vec{e}_j;\ \Gamma_{k,ij} = \vec{e}_{k,ij} = \vec{e}_k\partial_j\vec{e}_i. \quad (12)$$

Since our space is distorted, it is advisable to consider covariant differentiation (13):

$$D\vec{r} = \partial_k\vec{r}dx^k = \vec{e}_j\left(\partial_k x^j + x^i\Gamma_{ik}^{j}\right)dx^k = \vec{e}_j(D_k x^j)dx^k = \vec{e}_j Dx^j, \quad (13)$$

where Dx^j – covariant differential, $D_k x^j$ –covariant derivative, which for x_j will have the form (14):

$$D_k x_j = \partial_k x^j + x_i\Gamma_{jk}^{i}. \quad (14)$$

The change in the radius vector from the sequence of changes in the network state is defined as follows (15):

$$(D_m D_k - D_k D_m)x^i dx^k dx^m\vec{e}_i = R_{j,mk}^{i}x^j dx^k dx^m\vec{e}_i, \quad (15)$$

where $R_{j,mk}^{i}$ – Riemann tensor or curvature tensor, which is determined with the use of the Christoffel symbols (16):

$$R_{j,mk}^{i} = \partial_m\Gamma_{jk}^{i} - \partial_k\Gamma_{jm}^{i} + \Gamma_{pm}^{i}\Gamma_{jk}^{p} - \Gamma_{pk}^{i}\Gamma_{jm}^{p}. \quad (16)$$

(14) takes into consideration that (17):

$$(D_m D_k - D_k D_m)x^i = R_{j,mk}^{i}x^j. \quad (17)$$

The curvature tensor $R_{j,mk}^{i}$ can be collapsed by a pair of indices, obtaining a tensor of the second rank, for example, by the first and third indices. As a result, the Ricci tensor is obtained (18):

$$R_{jk} = R_{j,ik}^{i} = \partial_i\Gamma_{jk}^{i} - \partial_k\Gamma_{ji}^{i} + \Gamma_{pi}^{i}\Gamma_{jk}^{p} - \Gamma_{pk}^{i}\Gamma_{ji}^{p}. \quad (18)$$

Another contraction of this tensor leads to a scalar called the scalar space curvature (19):

$$R = g^{jk}R_{ik}. \quad (19)$$

Considering (18), the change in the symmetric metric tensor in Riemannian geometry can be determined with the use of the Ricci flow:

The authors consider certain options for determining the metric tensor. The author also note that the components of the metric tensor are determined from the following formula (20):

$$g_{ij} = \vec{e}_i\vec{e}_j = \cos\theta_g \quad (20)$$

where θ_g – angle between $\vec{e}_i$ and $\vec{e}_j$.

For Riemann geometry, the angle between x_i and x_j is determined with the use of g_{ij} (21):

$$\cos\theta_g = \frac{x_i x_{j_g}}{\sqrt{x_i x_{j_g} x_i x_{j_g}}} \quad (21)$$

In Euclidean space, to determine $\cos\theta_g$, the authors use the cosine theorem (22):

$$x'^2 = x_i^2 + x_j^2 + 2x_i x_j\cos\theta_g \quad (22)$$

hence (23):

$$\cos\theta_g = \frac{x_i^2 + x_j^2 - x'^2}{2x_i x_j} \quad (23)$$

To represent a hyperbolic space, it is proposed to use the Poincare disk, which is a unit disk on the complex plane z $|z| < 1, z = x + iy$ with the Riemannian metric (24):

$$ds^2 = \frac{4dzd\bar{z}}{(1 - z\bar{z})^2}. \quad (24)$$

When making a turn in hyperbolic space, the Moebius transform is used (25):

$$z \to exp^{i\theta}\frac{z - z_0}{1 - z\bar{z}_0} \quad (25)$$

In this case, the angles between the orts (components of the metric tensor) are determined by the discrete metric according to the law of hyperbolic cosine (26):

$$\cos\theta_g = \frac{coshx' - coshx_i coshx_j}{sinhx_i sinhx_j} \quad (26)$$

The Poincare disk model is used to calculate distances in hyperbolic space. The authors propose to represented the surface s by the set of points that satisfy the inequality $\sum_{i=1}^{n} x_i < 1$, where x_i is the coordinate, and n – is the dimension. Then the Riemann metric looks as follows (27):

$$ds^2 = \frac{4\sum_i^n x_i^2}{(1 - \sum_i^n x_i^2)^2}. \quad (27)$$

Let z_ω and z_ξ – be two points on the Poincare disk, according to the geodesic form, passing through z_ψ and z_ζ and intersecting the unit circle at z_ω and z_ξ, where z_ω is closer to z_ψ, and z_ξ is closer to z_ζ. The hyperbolic distance between z_ω and z_ξ is defined as follows (28):

$$d(z_\omega, z_\xi) = \left(ln\frac{(z_\omega - z_\psi)(z_\xi - z_\zeta)}{(z_\xi - z_\psi)(z_\omega - z_\zeta)}\right)^{-1} \quad (28)$$

Accordingly, the authors describe the Moebius transform for the Poincare disk as an offset (29):

$$F_c(z) = c + \frac{r^2}{\bar{z} + \bar{c}} \quad (29)$$

where c – circle centre; r – circle radius.

Thus, the network topology can be represented on a canonical unit disk with round holes, where the nodes are transformed to the form of virtual coordinates.

The authors suggest to consider the definition of the metric tensor for two cases, which are presented in Fig. 5.

Let the network comprise three nodes (Fig. 5a): A, B, and C. The authors note that there is a two-way connection between each pair of nodes, that is, $\overrightarrow{AB} \neq \overrightarrow{BA}$. The authors denote: $\overrightarrow{AB} = \vec{c}$ and $\overrightarrow{BA} = \vec{c}'$, since they are oppositely directed.

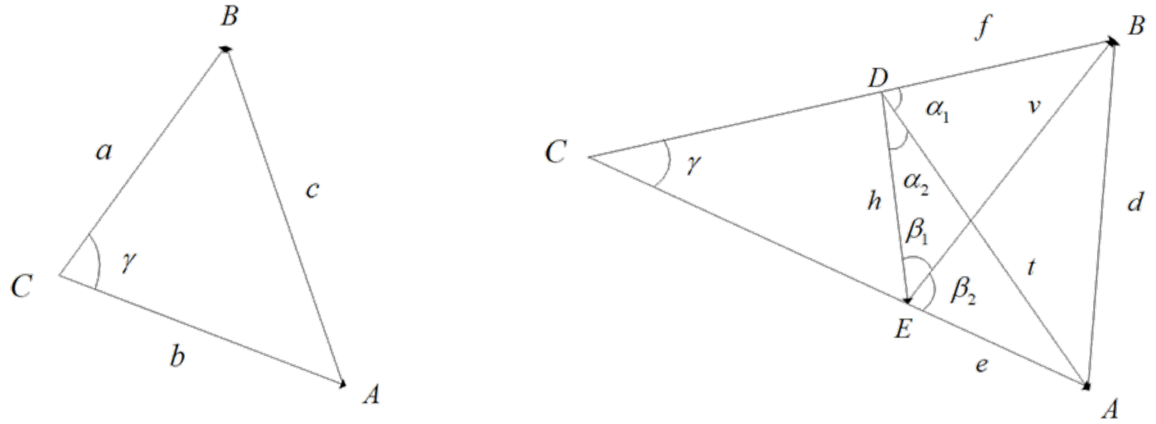

Figure 5. Determination of the angle between the vectors: a) when the point (input or output node) is common; b) when the vectors do not have common points

Source: developed by the authors

(30, 31):

$$g_{ab} = g_{a^*b^*} = \cos\gamma = \left(\vec{a}^2 + \vec{b}^2 - \vec{c}^2\right)/2\vec{a}\vec{b}, \quad (30)$$

$$g_{ab}{}^{*} = g_{a^*b} = \cos\gamma = \left(\vec{a}^2 + \vec{b}^2 - \vec{c}'^2\right)/2\vec{a}\vec{b}. \quad (31)$$

For (30), (31), the used indices a, a^* and b, b^* correspond to the vectors $\vec{a}$ and $\vec{b}$, respectively, and it is assumed that when a pair of indices does not simultaneously have the mark "*", then according to (27), x' corresponds to $\vec{c}$, otherwise $-\vec{c}'$.

The components of the metric tensor for the vectors $\vec{a}$ and $\vec{a}'$ will have the following form (Table 1).

Table 1. Components of the metric tensor for vectors $\vec{a}$ and $\vec{a}'$

	a	a^*	a'	a'^*
a	1	1	-1	-1
a^*	1	1	-1	-1
a'	-1	-1	1	1
a'^*	-1	-1	1	1

Source: developed by the authors

It is clear that the product of the component of the metric tensor of one vector by another is equal to unity ($\vec{e}_i^2 = 1$), and for oppositely directed: -1, for example (32):

$$g_{aa'} = (\vec{a}^2 + \vec{a}'^2 - (\vec{a} + \vec{a}')^2)/2\vec{a}\vec{a}' = -1. \quad (32)$$

The authors present the components of the metric tensor for the vectors $\vec{a}$ and $\vec{b}$ (Table 2).

Table 2. Components of the metric tensor for vectors $\vec{a}$ and $\vec{b}$

	b	b*	b′	b′*
a	$\left(\vec{a}^2 + \vec{b}^2 - \vec{c}^2\right)/2\vec{a}\vec{b}$	$\left(\vec{a}^2 + \vec{b}^2 - \vec{c}'^2\right)/2\vec{a}\vec{b}$	$\left(\vec{a}^2 + \vec{b}'^2 - \vec{c}^2\right)/2\vec{a}\vec{b}'$	$\left(\vec{a}^2 + \vec{b}'^2 - \vec{c}'^2\right)/2\vec{a}\vec{b}'$
a^*	$\left(\vec{a}^2 + \vec{b}^2 - \vec{c}'^2\right)/2\vec{a}'\vec{b}$	$\left(\vec{a}^2 + \vec{b}^2 - \vec{c}^2\right)/2\vec{a}\vec{b}$	$\left(\vec{a}^2 + \vec{b}'^2 - \vec{c}'^2\right)/2\vec{a}\vec{b}'$	$\left(\vec{a}^2 + \vec{b}'^2 - \vec{c}^2\right)/2\vec{a}\vec{b}'$
a'	$\left(\vec{a}'^2 + \vec{b}^2 - \vec{c}^2\right)/2\vec{a}'\vec{b}$	$\left(\vec{a}'^2 + \vec{b}^2 - \vec{c}^2\right)/2\vec{a}'\vec{b}$	$\left(\vec{a}'^2 + \vec{b}'^2 - \vec{c}^2\right)/2\vec{a}'\vec{b}'$	$\left(\vec{a}'^2 + \vec{b}'^2 - \vec{c}'^2\right)/2\vec{a}'\vec{b}'$
a'^*	$\left(\vec{a}'^2 + \vec{b}^2 - \vec{c}'^2\right)/2\vec{a}'\vec{b}$	$\left(\vec{a}'^2 + \vec{b}^2 - \vec{c}'^2\right)/2\vec{a}'\vec{b}$	$\left(\vec{a}'^2 + \vec{b}'^2 - \vec{c}'^2\right)/2\vec{a}'\vec{b}'$	$\left(\vec{a}'^2 + \vec{b}'^2 - \vec{c}^2\right)/2\vec{a}'\vec{b}'$

Source: developed by the authors

The authors propose to consider a network of four nodes: A, B, D, and E (Fig. 5b), for which a component of the metric tensor of the vectors $\vec{e}$ and $\vec{f}$, is defined, which do not have common points. Since they cannot be determined with the use of (32), for this we need to find the angle $\angle C$ by using the cosine theorem for quadrangles (33):

$$\cos\angle C = \left(\vec{d}^2 + \vec{h}^2 - \vec{v}^2 - \vec{t}^2\right)/2\vec{e}\vec{f}. \quad (33)$$

The authors present the components of the metric tensor for the vectors $\vec{e}$ and $\vec{f}$ (Table 3).

Table 3. Components of the metric tensor for the vectors $\vec{e}$ and $\vec{f}$

	f	f^*	f'	f'^*
e	$(\vec{d}^2+\vec{h}^2-\tilde{v}^2-\tilde{t}^2)/2\vec{e}\vec{f}$	$(\vec{d}'^2+\vec{h}^2-\tilde{v}^2-\tilde{t}^2)/2\vec{e}\vec{f}$	$(\vec{d}^2+\vec{h}^2-\tilde{v}^2-\tilde{t}^2)/2\vec{e}\vec{f}'$	$(\overrightarrow{d'}^2+\vec{h}^2-\tilde{v}^2-\tilde{t}^2)/2\vec{e}\vec{f}'$
e^*	$(\vec{d}^2+\vec{h}'^2-\tilde{v}^2-\tilde{t}^2)/2\vec{e}\vec{f}$	$(\vec{d}'^2+\vec{h}'^2-\tilde{v}^2-\tilde{t}^2)/2\vec{e}\vec{f}$	$(\vec{d}^2+\vec{h}'^2-\tilde{v}^2-\tilde{t}^2)/2\vec{e}\vec{f}'$	$(\overrightarrow{d'}^2+\vec{h}'^2-\tilde{v}^2-\tilde{t}^2)/2\vec{e}\vec{f}'$
e'	$(\vec{d}^2+\vec{h}^2-\tilde{v}^2-\tilde{t}^2)/2\overrightarrow{\vec{e}'f}$	$(\vec{d}'^2+\vec{h}^2-\tilde{v}^2-\tilde{t}^2)/2\overrightarrow{\vec{e}'f}$	$(\vec{d}^2+\vec{h}^2-\tilde{v}^2-\tilde{t}^2)/2\overrightarrow{\vec{e}'f'}$	$(\vec{d}'^2+\vec{h}^2-\tilde{v}^2-\tilde{t}^2)/2\overrightarrow{\vec{e}'f'}$
e'^*	$(\vec{d}^2+\vec{h}'^2-\tilde{v}^2-\tilde{t}^2)/2\overrightarrow{\vec{e}'f}$	$(\vec{d}'^2+\vec{h}'^2-\tilde{v}^2-\tilde{t}^2)/2\overrightarrow{\vec{e}'f}$	$(\vec{d}^2+\vec{h}'^2-\tilde{v}^2-\tilde{t}^2)/2\overrightarrow{\vec{e}'f'}$	$(\vec{d}'^2+\vec{h}'^2-\tilde{v}^2-\tilde{t}^2)/2\overrightarrow{\vec{e}'f''}$

Source: comped by the authors

Where it is assumed that upon the use of the indices f^* and f'^* in (33), instead of $\vec{h}$ the authors use $\vec{h}'$, and for the indices e^* and e'^* in (32) instead of $\vec{d}-\vec{d}'$. It is also accepted that diagonal components are defined as follows (34, 35):

$$\tilde{v} = (\vec{v} + \vec{v}')/2, \tag{34}$$

$$\tilde{t} = (\vec{t} + \vec{t}')/2. \tag{35}$$

To determine the metric contravariant tensor, the following correlation is used (36):

$$g^{ij} = \frac{1}{g}\frac{\partial g}{\partial g_{ij}}, \tag{36}$$

where $g = det|g_{ij}|$ – determinant of metric covariant tensors.

The authors suggest to consider the case when the loads between nodes can be described with the use of the exponential distribution law. In particular, according to Fig. 5a, the load between the network clusters can be represented as follows:

$$CB: 1 - e^{-at}$$
$$CA: 1 - e^{-bt}$$
$$AB: 1 - e^{-ct}$$

From (33), the components of the metric tensor g_{ab} will be determined as follows (37):

$$g_{ab} = \frac{(1-e^{-at})^2 + (1-e^{-bt})^2 - (1-e^{-ct})^2}{2(1-e^{-at})(1-e^{-bt})}. \tag{37}$$

For (32), accordingly (38):

$$\frac{dg_{ab}}{dt} = \frac{\left((1-e^{-at})^2 + (1-e^{-bt})^2 - (1-e^{-ct})^2\right)}{2(1-e^{-at})(1-e^{-bt})}\left[\frac{ae^{-at}}{(1-e^{-at})} + \frac{be^{-bt}}{(1-e^{-bt})}\right] + \\ + \frac{ae^{-at}(1-e^{-at}) + be^{-bt}(1-e^{-bt}) - ce^{-ct}(1-e^{-ct})}{2(1-e^{-at})(1-e^{-bt})}. \tag{38}$$

Conclusions

Due to the high density of users, an important aspect is the development of a technique for placing sensor nodes, which provides the necessary coverage of the educational space. The section has improved the method of clustering the educational space to reduce the duration of the search for a route between any pair of them, which provides for the determination of the centroid of the cluster with consideration of the distribution model of certification within the educational space. The essence of the algorithm is to split the set of elements of the vector space into a predetermined number of clusters. At each iteration, the centroid is recalculated for each cluster obtained in the previous step, then the vectors are again divided into clusters according to which of the new centres turned out to be closer in metric, which is formed not only based on the calculation of the Euclidean distance between nodes and factoring in the propagation model of the educational space. It is proposed to use the Motley-Keenan peer-to-peer model as an estimate of losses in the studied educational distributed network, which allows to factor in losses in the level of assimilation of material and track the level of the educational distributed network upon passing through each network node. The formation of the cluster and, accordingly, the finding of its centroid will occur until all nodes closest in distance with the maximum value of students' perception of material are found.

To determine the state of a distributed educational network, it is proposed to use a model of tensor representation of its topological structure based on a curved coordinate system. It is proposed to increase the number of components of the metric tensor to represent the metric in a symmetric tensor field, which is used to describe the Riemann-metric strain, which is used in Ricci flows. The use of this model makes it possible to monitor and record the states of the topological structure after clustering, based on which orientation models will work, and also to increase the accuracy of the formation of the tables of formation of academic performance and perception of the material at specific points in time.

The prospects for further research are monitoring to determine the main factors for assessing the quality of distance education among employers, experienced professionals, and experts involved in the development, content, modification, administration of such courses, and monitoring the level of preparedness of professionals who have gained their qualifications through distance learning.

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Conflict of Interest

None.

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Міждисциплінарне застосування теорії ймовірностей та математичної статистики у професійній орієнтації фізиків та математиків

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Анотація

Актуальність. Методи математичної статистики можуть відігравати важливу роль у навчанні учнів, оцінюючи їхні математичні здібності, спрямовуючи вибір професійної орієнтації та оцінюючи потенційні рішення.

Мета. Метою дослідження є вивчення можливості використання повсюдно поширеної дистанційної освіти для спільної групової роботи з математичною підтримкою. Запропонований підхід передбачає використання кластерних та мережевих технологій, де кожен учасник виступає в ролі хоста для навчальної програми.

Методика. Методологія передбачає визначення ефективності застосованих методів шляхом оцінки потенціалу групового застосування методичного апарату та виявлення напрямів допомоги учням у виборі професійної орієнтації.

Результати. Результати демонструють, що у вищій школі використання математичного інструментарію дозволяє студентам розвивати здібності до прийняття рішень, оцінювати умови вибору, обирати навчальні курси та визначати галузі знань, в яких вони можуть досягти успіху. Для студентів-математиків такий підхід покращує їхню здатність застосовувати математичні концепції до практичних проблем через спільну групову роботу.

Висновки. Практичне значення дослідження визначається потенціалом розробленої програми для формування професійно підготовлених випускників.

Ключові слова: університет; теорія ймовірностей; статистика; освіта; професія.